How to Write a Proof

d.PotD Guide

1 What is a Proof?

A proof is like a detective's case file. You aren't just saying, "The butler did it" (the answer); you are showing all the evidence and logical steps that prove the butler had to do it.

For d.PotD, your proof will explain how you got to your formula or result and prove that this is the correct formula or result.

2 The Four Parts of a Strong Argument

Every strong proof or argument needs these parts.

1. **The Claim:** This is the statement you want to prove. (Example: "My machine will always sort the balls in less than 5 steps.")
2. **What You Know (Rules):** These are the facts, definitions, or rules you are allowed to use. (Example: "The balls only move left or right," or "An even number is any number divisible by 2.")
3. **The Steps (The Logic):** This is the main part. It's a list of clear, logical steps that connect what you know to your claim. Each step must make sense!
4. **The Conclusion:** A simple sentence restating that you have proven your claim.

3 Three Simple Ways to Prove Your Claim

Depending on the problem, one of these methods will work best.

3.1 Direct Proof: The "Step-by-Step" Method

This is the simplest method. You start with the rules and follow a straight path of logic until you reach your answer.

Example 1 (Direct Proof Structure: Adding Odd Numbers)**. Claim:** If you add any two odd numbers, the result is always an even number.

1. **What We Know:** An odd number is 1 more than an even number. We can write an odd number as $2 \times (\text{something}) + 1$.
2. Let the first odd number be $A = (2 \times k) + 1$.

3. Let the second odd number be $B = (2 \times m) + 1$ (where k and m are just any whole numbers).

4. We add them up:
$$A + B = (2k + 1) + (2m + 1)$$

5. Group the parts together:
$$A + B = 2k + 2m + 2$$

6. We can factor out a 2 (since everything is multiplied by 2):
$$A + B = 2 \times (k + m + 1)$$

7. **Conclusion:** Since the final answer is $2 \times$ (a whole number), the result is an even number.

3.2 Proof by Contradiction: The "Impossible" Method

Sometimes it's easier to prove something is true by showing that the opposite is **impossible**.

1. Assume your claim is **false** (assume the opposite is true).

2. Follow the logic of this false assumption.

3. If the logic leads to a crazy, **impossible** answer (a **contradiction**), then your first assumption must be wrong, and your original claim must be true!

Example 2 (Contradiction: Birthday Month Logic)**. Claim:** In any group of 13 or more people, at least two people must share the same birth month.

1. **Assume the Opposite:** Let's assume the claim is **false**. We assume that in a group of 13 people, no two people share the same birth month.

2. **Apply the Rules:** We know there are only 12 months in a year.

3. If each of the 13 people has a different birth month, we would need 13 different months.

4. **Contradiction:** This is impossible! We only have 12 months. Since our assumption led to an impossible situation, the assumption was wrong.

Conclusion: The original claim must be true: at least two people share a birth month.

3.3 Proof by Induction: The "Domino Effect" Method

This method is perfect for problems that involve a repeating pattern or a long sequence (like proving a design works for N balls, N steps, or N trials). It works like knocking over dominos.

1. **Base Case (The First Domino):** You show that your claim works for the very first step, $N = 1$. (Example: "My machine works when there is just 1 ball.")
2. **Inductive Hypothesis (The Chain):** You **assume** that if the claim works for any number of steps k, it will also work. (Example: "Let's assume the machine works perfectly for k balls.")
3. **Inductive Step (The Tipping Point):** You use your assumption about k to prove that it **must** work for the next number, $k + 1$. (Example: "If it works for k balls, and we show how the $(k + 1)$-th ball is correctly handled, then it must work for all balls.")

If you prove the first domino falls (Base Case) and that one domino falling always knocks over the next one (Inductive Step), then all the dominos must fall!